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Total Pages: 2

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APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION, DECEMBER 2017

Course Code: MA201

Course Name: LINEAR ALGEBRA AND COMPLEX ANALYSIS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer any two full questions, each carries 15 marks. Marks

1 a) Find the points where Cauchy-Riemann equations are satisfied for the function $f(z) = xy^2 + i\,x^2 y$. Where does $f^{|}(z)$ exist? Is the function f(z) analytic at those points? (7)

b) If $v = e^x (x \sin y + y \cos y)$, find an analytic function f(z)=u+iv. (8)

2 a) Show that $u = x^2-y^2-y$ is harmonic. Also find the corresponding conjugate harmonic function. (7)

b) (i) Find a bilinear transformation which maps $(-i, 0, i)$ onto $(0, -1, \infty)$. (8)

(ii) Test the continuity at z = 0, if $f(z) = \frac{Im\, z}{|z|}, z \neq 0$

$= 0, z = 0$

3 a) Find the image of the lines x=1, y=2 and x>0, y<0 under the mapping $W= z^2$ (8)

b) Find the image of the semi-infinite strip $x > 0$, $0< y < 2$ under the transformation w=iz+1. Draw the regions. (7)

PART B

Answer any two full questions, each carries 15 marks.

4 a) Evaluate $\oint Re\, z^2 dz$ over the boundary C of the square with vertices 0, i, 1+ i,1 clockwise (8)

b) Evaluate $\int \frac{4-3z}{z(z-1)}$ dz over the circle $|z|=\frac{3}{2}$ (4)

c) Evaluate $\int \frac{3z^2+7z+1}{z+1}$ dz over the circle |z+ i |=1 (3)

5 a) Expand $\frac{z}{(z-1)(z-2)}$ in (1) 0<|z-2|<1, (2) |z-1|>1 (8)

b) Evaluate $\int_0^{2\pi} \frac{1}{2+\cos\theta}\, d\theta$ (7)

6 a) Using Residue theorem evaluate $\int \frac{z^2}{(z-1)^2(z+2)}$ dz over the circle |z|=3 (7)

b) Find the Taylor series of $\frac{\sin z}{z-\pi}$ about the point $z= \pi$ (4)

c) Evaluate $\int \frac{\sin z}{z^6}$ dz over the circle |z|=2 using Cauchy's Residue theorem. (4)

PART C

Answer any two full questions, each carries 20 marks.

7 a) Solve by Gauss-Elimination method x + y + z = 6, x+ 2y- 3z = -4, -x-4y+9z =18. (7)

b) Find the values of 'a' and 'b' for which the system of equations x + y + 2z =2, 2x-y+3z=10,5x-y+az=b has: (7)

(i) no solution (ii) unique solution (iii) infinite number of solutions.

c) Verify whether the vectors (1,2,1,2), (3,1,-2,1),(4,-3,-1,3) and (2,4,2,4) are linearly independent in R^4 . (6)

8 a) Write down the matrix associated with the quadratic form $8x_1^2+7x_2^2+3x_3^2-12x_1x_2-8x_2x_3+4x_3x_1$. By finding eigen values, determine nature of the quadratic form. (7)

b) Diagonalise the matrix A = $\begin{bmatrix} 1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & -1 \end{bmatrix}$ (7)

c) If A is a symmetric matrix, verify whether AA^T and A^TA are symmetric? (6)

9 a) Find the eigen vectors of A = $\begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$ (8)

b) Find the null space of AX=0 if A= $\begin{bmatrix} 1 & 1 & 0 & 2 \\ -2 & -2 & 1 & -5 \\ 1 & 1 & -1 & 3 \\ 4 & 4 & -1 & 9 \end{bmatrix}$ (6)

c) Verify whether $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal. (6)

What can you say about determinant of an orthogonal matrix? Prove or disprove the result.

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